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Full length article

MATHEMATICAL APPROACH TO FORECAST THE ELECTRICITY CRISES IN PAKISTAN AND ITS MITIGATION

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ABSTRACT

Electricity is widely recognized as the backbone of economic success and progress, playing a vital role in driving socioeconomic development. This comprehensive study conducted an in-depth analysis of Pakistan's electricity consumption and aimed to develop a suitable model for forecasting the country's future electricity consumption (E.C.). Due to the strong positive correlation between E.C. of different sectors with Population and Gross Domestic Product (GDP), multiple linear regressions were employed for the analysis of electricity consumption. To predict population and GDP different mathematical model such as linear, Exponential, Polynomial and logarithmic models were applied on population and GDP data. In order to get the best fitted model a number of goodness-of-fit tests Coefficient of Determination, Sum of Square Errors, Sums of Square Regression, Mean Square Error, Root Mean Square Error, And F- Statistics (Adj-R^2, SSE, SSR, MSE, RMSE and F Statistics) applied and got the polynomial model to be the most effective one for population and GDP. According to the goodness-of-fit test-based models, Pakistan's population is on track for steady growth, with projections reaching approximately 244428310 in 2025 and further increasing to 319825879 in 2040. This indicates significant demographic changes on the horizon. At the same time, Pakistan's GDP per capita is expected to rise, starting at 1958.3 $ in 2025 and potentially reaching 3,210.9 $ in 2040, signifying a positive trajectory towards economic development and prosperity in the country. Based on the selected models suggested by goodness of fit tests, the study throws light on the expected future electricity consumption in Pakistan. According to these projections, the expected electricity consumption of Pakistan may reach 121,121.1 GWh in 2025, 138,861.8 GWh in 2030, 157,716.8 GWh in 2035, and 177,686.3 GWh in 2040 respectively.

KEYWORDS: Energy in Pakistan, Economic Growth, Power Crises, Demand Forecasting.

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1. INTRODUCTION

Just like food, water, and air, electricity has become a basic life need. People use electricity for heating, lighting, and freezing and run various electrical equipment, including computers and electronic machines. Electricity can also play an important role in a country's economy.

In modern life, electricity plays an important role in different industries, such as mining,

manufacturing, communication, and transportation. Electricity is considered the backbone of the economy's success and growth, so it plays a key role in socioeconomic development [1]. For goods and services, electricity production and consumption play a significant role. The literature says that different types of factors have a direct effect on the country's electricity demand. Some of them are a growing population, extensive urbanization, industrialization of economies, and the increasing use of electrical appliances in daily life, all of which have directly contributed to the increase in demand for electricity. Electricity is one of the oldest forms of energy, but it is still a very important part of our lives today. It has helped us in many ways and grown in many areas. Most of the technology we use every day needs electricity to work. The total amount of electricity we need depends on the temperature [2], wind speed, rain, etc., and everyday activities [3]. The high temperature also causes more electricity to be used by air conditioners and other electrical appliances [4]. In 2008, Pakistan is said to have generated about 11,500 MW per day of electricity. In 2010, the economy will use about 20,000 MW per day of electricity [3].

During the past 30 years, electricity supply and consumption have grown by about 40 times [5]. Any country's economic situation depends on the availability of electricity; currently, Pakistan is severely affected by a lack of electricity, so the country's economic growth is directly affected by the lack of electricity [6]. The population growth rate of 5.81% is still short of the 6.6% required to overcome unemployment and poverty rates, which are challenges for the current electrical crisis as compared to Pakistan's GDP growth, and it is severely affecting all sectors of Pakistan [7].

The population growth rate, which is expected to be around 3% per year, is driving up the need for electricity. More resources like natural gas, lubricant, and melted petroleum gas are being used to keep up with the growing demand for electricity in the country. The electrical industry uses about 38.8 million Tons of Oil each year (MTOE), as reported by the Pakistan economic survey [8]. According to a study by the World Bank, a country in the world needs to improve its financial system with the best technology to make electricity [9]. Our country's electricity production sources are hydropower, oil, natural gas, nuclear power, and other forms of energy. The key reasons behind the electricity deficit for this are the recycling balance, the economy, and the demand and use of energy. Hydro energy and coal resources are the primary drivers of energy shortages and insufficient electricity output. They provide little power but are inexpensive and highly polluting [10]. In South Asia, Pakistan is the most developing and agricultural country. A capacity of 60 M.W. was installed when Pakistan was established [11]. In 1980 the capacity of generation was 3.76 million K.W but the consumption of electricity was 10.16 million K.W. at that time, the population of Pakistan was around 80 million. In 2018 the population increased to 197 million, and electrical energy consumption also increased to 98,500 GWh. At that time, the installed

generation capacity was only 29,953 MWh, but production was 116,050 GWh [7]. However, still, a wider gap between consumption and installed capacity than there was previously.in 2015 the electricity consumption deficit reached 7000 MW [12]. In summer, Daily power outages of roughly 12 hours in the urban and rural regions due to this being affected, many businesses have been forced to stop or reduce their operation [13'14, 19]. The increase in population, weak governess, and the poor condition of the power plant are some major factors of power crises in Pakistan [15]. Literature also shows that various mathematical models have been used to minimize these problems. Pakistan's total electricity consumption and components were forecasted using Holt-Winters and ARIMA models [16'17'20'30]. To estimate China's total energy output and consumption, [18] employed the optimized single variable discrete grey Forecasting model (ODGM) and QP-Markov technique. [19] Used an optimized grey model (OGM) (1, 1) to anticipate Turkey's power usage. The Grey Prediction Model finds application in simulating and predicting outcomes using limited data samples. At the foundation of the grey prediction system is the GM (1, 1) model, which serves as its essential and primary component. This model has gained extensive utilization across various domains of life, including industry, energy, environment, and the prediction of natural disasters [22'23]. Forecasting electricity consumption is vital for the electric power sector, guiding supply strategies. Maintaining a balance between consumption and production is challenging due to electricity's non-storability and transport limitations. The stochastic nature of consumption adds complexity to accurate prediction [24]. A study was conducted to examine how economic and demographic factors impact Italy's yearly electricity consumption. The goal was to create a predictive model for long-term consumption forecasting [25, 31, 32].

2. DATA DESCRIPTION AND METHODOLOGY

Electricity consumption depends on different parameters, such as a country's economy, temperature, and population. For this analysis, the gross domestic product (GDP) and population data from 1970–2020 have been taken from [World Bank, 2022 open data]. Electricity consumption statistics (household, commercial, industrial, agricultural, streetlight and other) have been taken from the Federal Bureau of Statistics (FBS).

Figure [1] depicts that all the parameters of electricity consumption have a strongly positive trend over time. The decrease in consumption in 2019 and 2020 is likely due to the economic slowdown caused by the COVID-19 pandemic, which affected numerous sectors of the economy. On the other hand, the rapid increase in household EC is likely due to the increased use of electricity for household activities during the lockdown period.

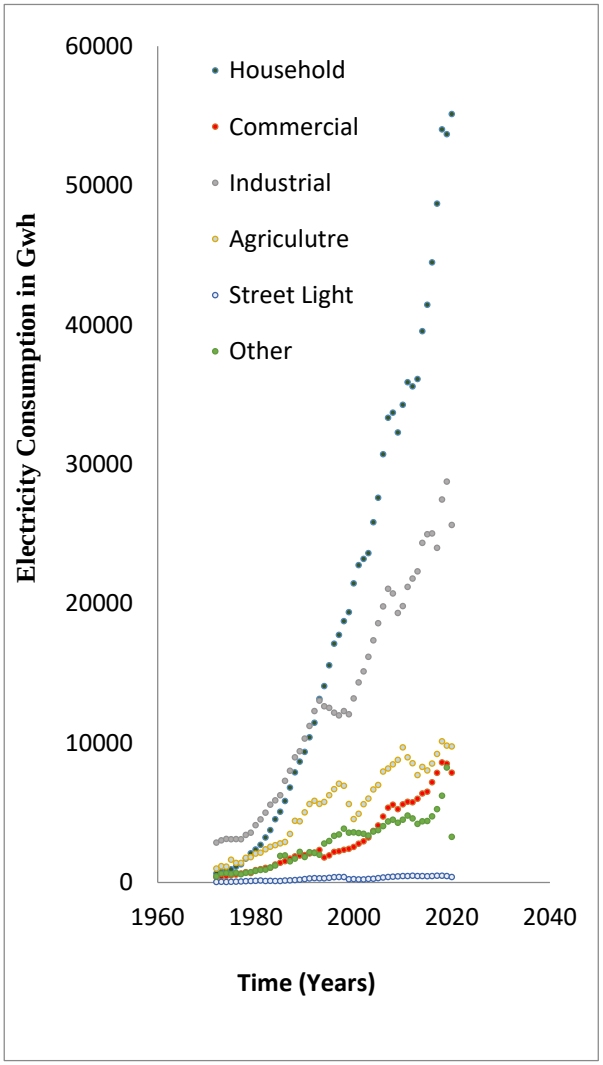

Figure 1: Historical data of sector wise electricity consumption of Pakistan from 1970-2020.

The data of the population for the same period is reported in Figure 2, and it has an increasing trend throughout the considered period.

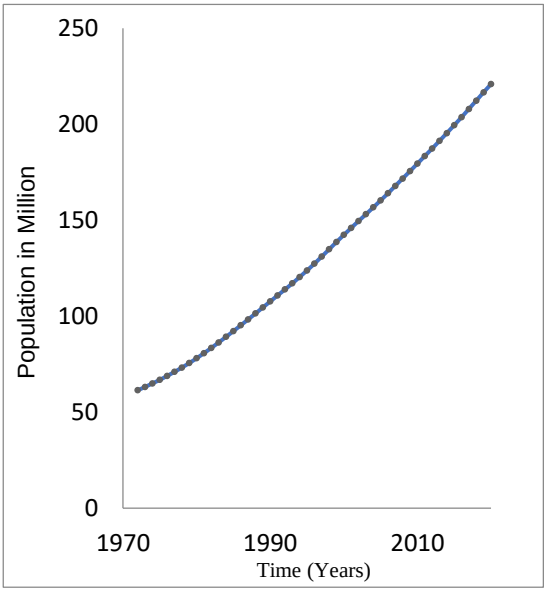

Figure 2: Historical data for the Population of Pakistan from 1972-2020.

The demographic variables (i.e., GDP per capita) are presented in Fig 3. Data collected from the World Bank data has interesting behavior, slightly increasing up to 2002, and the increase in GDP per capita from 2003 to 2018 highlights the growing economic activity in the country and the corresponding increase in demand for electricity. The sudden changes in the GDP per capita in 2018, when it reached a peak value, can be attributed to the fluctuations in oil and gas prices. On the other hand, the rapid decrease in GDP per capita in 2019 and 2020 is likely due to the economic slowdown caused by the COVID-19 pandemic and the associated lockdown measures. This highlights the impact that economic events and shocks can have on the demand for electricity.

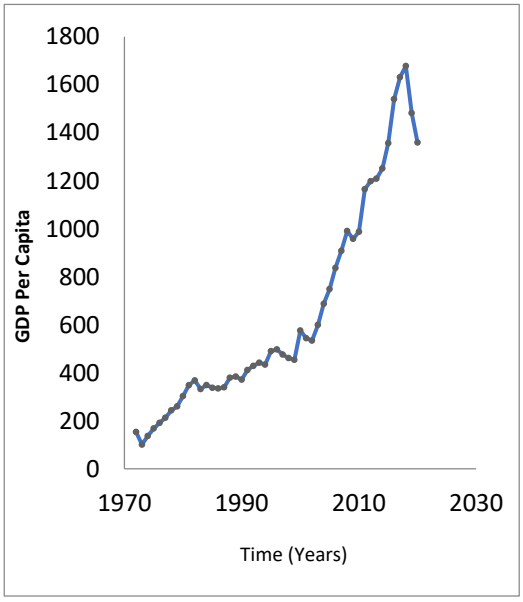

Figure 3: Historical data for the GDP 1972-2020.

3. PREDICTION METHODS

The proposed models are designed to help us understand how different factors, such as population, GDP per capita, and others, affect electricity consumption in various sectors. By analyzing these relationships, the models can give us a more complete view of how electricity consumption is influenced by different factors. To forecast population and GDP used different models, including linear, exponential, quadratic, and logarithmic models [26'27'28] and chose the model that provided the best fit to the data, which helps to improve the accuracy of predictions.

The main purpose of this article is to deeply analyze the EC of Pakistan and try to get a suitable model to forecast the EC of different sectors in the future. The first model linking annual household electricity consumption to population and GDP per capita is particularly useful for understanding the demand for residential electricity. The second model linking commercial electricity consumption to population and GDP provides insights into the demand for commercial electricity. The third model, which links industrial electricity consumption to population, and GDP, provides a more nuanced view of the demand for industrial electricity, taking into account the impact of technical progress. The other three models for agricultural, streetlight and other uses provide a more detailed view of the demand for electricity in specific sectors. The seventh model for total electricity consumption provides an overall picture of the demand for electricity in the country. In conclusion, the proposed models provide a complete view of the factors that affect EC in Pakistan. By using these models, it is possible to make informed decisions about electricity generation and distribution that take into account the complex relationships between different factors.

4. MULTIPLE LINEAR REGRESSIONS

Multiple linear regressions are a statistical method used to model the relationship between a dependent variable and multiple independent variables. In this method, a multiple linear equation is used to predict the value of the dependent variable based on the values of the independent variables. Generally, the longer the data set, the more accurate the final result [29].

The multiple linear regression trend models are generally expressed as follows:

$$y = \alpha + b_1 x_1 + b_2 x_2$$

y: Dependent variable

α Constant parameter of model

b_1, b_2 Coefficient of independent variables

x_1, x_2 Independent variables

5. CORRELATION BETWEEN DIFFERENT SECTORS OF ELECTRICITY CONSUMPTION WITH POPULATION AND GDP

The correlation between electricity consumption in different sectors (Household, commercial, industrial, Agriculture streetlight and others) with population (Iqbal et al, 2023) and GDP are given below in table.

Table 1: correlation b/w electricity with population and GDP

Correlation	pop	GDP
Household	0.985	0.965
Commercial	0.965	0.982
Industrial	0.985	0.944
Agriculture	0.944	0.869
Street light	0.902	0.85
Other	0.938	0.882
Total	0.989	0.957

6. MODEL OF DIFFERENT PARAMETERS

The models of electricity consumption in different sectors (Household, commercial, industrial, Agriculture streetlight and others are given below in table.

Table 2: Mathematical Model of Different Sectors of Electricity Consumption

Model	Equation
Household	$H(EC) = -19064 + 0.000238P + 11.3GDP$
Commercial	$C(EC) = -1366 + 0.000016P + 3.83GDP$
Industrial	$I(EC) = -7090 + 0.000147P + 1.47GDP$
Agriculture	$A(EC) = -2783 + 0.000071P - 1.77GDP$
Street light	$St\ L(EC) = -121 + 0.000003P - 0.0146GDP$
Other	$O(EC) = -1810 + 0.000038P - 0.536GDP$

7. ERROR ANALYSIS

Models of different sectors check by using of different goodness of test such as Coefficient of Determination, Sum of Square Errors, Sums of Square Regression, Mean Square Error, Root Mean Square Error, and F- Statistics (R^2, SSE, SSR, MSE, RMSE and F Statistics) as shown in Table 3a and 3b.

Table 3a: Goodness of Fit test for different sectors of electricity consumptions.

Model	Adj-R^2	SSR	SSE
Household	98.40%	1.28E+10	2E+08
Commercial	98.30%	2.82E+08	4602518
Industrial	98.10%	2.9E+09	53069630
Agriculture	91.60%	3.57E+08	31143444
Street light	84.40%	922189	162132
Other	86.70%	1.27E+08	18486954
Population	98.80%	4.7E+10	5.57E+08
GDP	95.05%	2619396	14201539

Table 3b: Goodness of Fit test for different sectors of electricity consumptions.

Model	MSE	F values	RMSE
Household	433745	1475.49	2017.89
Commercial	100055	1407.52	306.47
Industrial	1153688	1255.4	1040.69
Agriculture	677031	263.96	797.23
Street light	3525	130.82	57.52
Other	401890	157.51	614.23
Population	12118185	1940.72	3372.87
GDP	55731.83	254.81	538.35

8. BEST FITTED MODEL FOR POPULATION

To forecast population different mathematical model have been applied to data (linear, exponential, polynomial and logarithmic model) [26, 27, 31, 32]. Based on goodness of fit test polynomial is the best fitted model as shown in table 4a and 4b.

Table 4a: checking of model by different goodness of test for GDP.

	Polynomial	Exponential
Model	$y = 0.751656143x^2 - 2971.97612x + 2937949.971$	$y = 2E - 39e^{0.0479x}$
Adj-R^2	95.69	95.05
SSR	8920546	15845073
SSE	401638.9	2619396
MSE	8196.712	53457.06
F values	1088.308	296.4075
RMSE	90.53569	231.2078

Table 4b: checking of model by different goodness of test for GDP.

	Linear	Logarithmic
Model	$y = - 56510 + 28.6x$	$y = 57094\ln(x) - 433201$
Adj-R^2	85.9	86.02
SSR	8259322	7.45E+14
SSE	1529764	7.45E+14
MSE	31219.67	1.58E+13
F values	264.555	47.00001
RMSE	176.6909	3980305

9. BEST FITTED MODEL FOR GDP

To forecast GDP different mathematical model have been applied to data (linear, exponential, polynomial and logarithmic model) [31, 32]. Based on goodness of fit test polynomial is the best fitted model as shown in table 5a and 5b.

Table 5a: checking of model by different goodness of test for population.

	Polynomial	Exponential
Model	$y = 22523x2 - 86530751x + 83109550085$	$y = 4E - 16e^{0.271x}$
Adj-R^2	99.98	0.9934
SSR	1.13E+17	1.59E+17
SSE	2.54E+13	3.21E+13
MSE	5.19E+11	6.54E+11
F-values	217471.7	171420.1
RMSE	720539	808774.8

Table 5b: checking of model by different goodness of test for population.

	Linear	Logarithmic
Model	$y = - 6.62E + 09 + 3382310x$	$y = 7E + 09 \ln(x) - 5E + 10$
Adj-R^2	99.3	0.9922
SSR	1.12E+17	4.59E+20
SSE	8.53E+14	5.01E+20
MSE	1.74E+13	1.02E+19
F-values	6442.996	44.92
RMSE	4172020	3.06E+09

10. FORECASTED RESULT

The projected outcomes of GDP, Population and household EC are presented in table 6 for the time period between 2024 and 2040.

Table 6: forecasting values of population, GDP and household EC from 2024-2040.

years	GDP	Pop	Household
2024	1886.82	239762179	59320.46
2025	1958.299	244428310	61238.72
2026	2031.282	249139488	63184.69
2027	2105.769	253895713	65158.36

2028	2181.758	258696985	67159.75
2029	2259.251	263543303	69188.84
2030	2338.247	268434668	71245.64
2031	2418.747	273371079	73330.15
2032	2500.749	278352537	75442.37
2033	2584.255	283379042	77582.3
2034	2669.265	288450593	79749.93
2035	2755.778	293567191	81945.28
2036	2843.794	298728835	84168.33
2037	2933.313	303935526	86419.09
2038	3024.336	309187264	88697.56
2039	3116.862	314484048	91003.74
2040	3210.891	319825879	93337.63

Forecasted results of commercial, industrial and Agriculture from 2024 to 2040 are given below in table 7.

Table 7: forecasting of commercial, agriculture and industrial sectors from 2024-2040.

years	Commercial	Industrial	Agriculture
2024	9696.71	30928.67	10900.44
2025	10045.14	31719.66	11105.22
2026	10400.04	32519.49	11310.53
2027	10761.42	33328.15	11516.39
2028	11129.29	34145.64	11722.77
2029	11503.62	34971.96	11929.7
2030	11884.44	35807.12	12137.16
2031	12271.74	36651.11	12345.17
2032	12665.51	37503.92	12553.7
2033	13065.76	38365.57	12762.78
2034	13472.49	39236.06	12972.39
2035	13885.7	40115.37	13182.54
2036	14305.39	41003.52	13393.23
2037	14731.56	41900.49	13604.46
2038	15164.2	42806.3	13816.22
2039	15603.32	43720.94	14028.52
2040	16048.93	44644.41	14241.36

Forecasted results of streetlight and others sectors of electricity consumption from 2024 to 2040 are given below in table 8.

Table 8: forecasting of streetlight and others sectors from 2024-2040.

Years	Street light	Other	Total
2024	570.739	6289.627	117706.7
2025	583.6938	6428.627	121121.1
2026	596.7617	6568.533	124580.1
2027	609.9429	6709.345	128083.6
2028	623.2373	6851.063	131631.7
2029	636.6448	6993.687	135224.5
2030	650.1656	7137.217	138861.8
2031	663.7995	7281.653	142543.6
2032	677.5467	7426.995	146270.1
2033	691.407	7573.243	150041.1
2034	705.3805	7720.397	153856.7
2035	719.4672	7868.456	157716.8
2036	733.6671	8017.422	161621.6
2037	747.9802	8167.294	165570.9
2038	762.4065	8318.072	169564.8
2039	776.946	8469.756	173603.2
2040	791.5986	8622.346	177686.3

11. RESULTS AND DISCUSSION

Using regression analysis, we found that the suggested models correctly predicted how electricity would be used in the future in a number of Pakistani electricity sectors. It was found that the coefficients of the independent variables had a significant effect on how much electricity was used. This shows how different factors affect demand for electricity.

In terms of the coefficients for population, we still see that the household, commercial, and industrial sectors have the highest coefficients, indicating that a change in the population will have a greater impact on electricity consumption for these sectors. Similarly, the street light sector still has the lowest coefficient, suggesting that changes in the price of

electricity will have a smaller impact on electricity consumption for this sector.

Looking at the coefficients for GDP, we see that the household sector still has the highest coefficient, indicating that changes in GDP will have a larger impact on electricity consumption for this sector compared to the other sectors. The agriculture sector still has a negative coefficient, suggesting that an increase in GDP will lead to a decrease in electricity consumption for this sector.

The regression results for the first model, which linked annual household EC to population and GDP per capita, showed that both population and GDP per capita had positive effects on household EC. Both variables were found to have statistically significant correlations, which means that a rise in population and GDP will likely lead to a rise in the amount of electricity that households use. In a second model that linked GDP and population, both were found to have a positive effect on how much commercial energy was used. There was a statistically significant link between population growth and GDP growth, which suggested that business electricity use would also go up. The results of the third model, which linked industrial electricity consumption to population, and GDP, indicated that population and GDP, had a positive effect on industrial electricity consumption. All of the links between the two factors were statistically significant, which shows that more people, more money, and more advanced technology will all lead to increased industrial energy use. The results of the other three models for agricultural, streetlight, and other uses showed similar patterns, with positive impacts on population and a negative impact on GDP from electricity consumption in those sectors. The results of the seventh model for total electricity consumption showed that all independent variables had positive impacts on total electricity consumption, providing further evidence of the complex relationships between different factors and the demand for electricity in Pakistan.

Figures 4 demonstrate a forecasted upward trajectory, signaling a rise in electricity consumption over time across various sectors: households, industrial, and agriculture sectors.

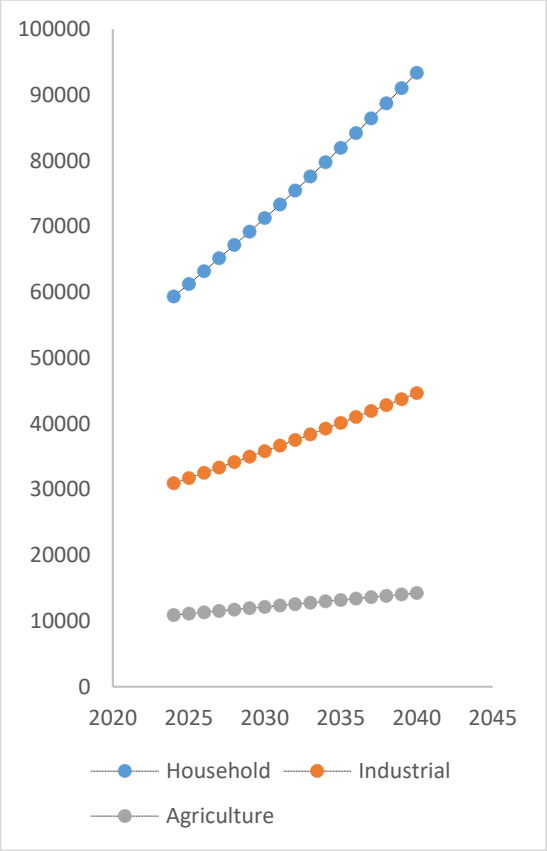

Figure 4: Graphical anaylsis of forecasted values from 2024-2040 of Household,industrail and agricluture sectors.

Figures 5 demonstrate a forecasted upward trajectory, signaling a rise in electricity

consumption over time across various sectors: commercial, streetlight, and others sectors.

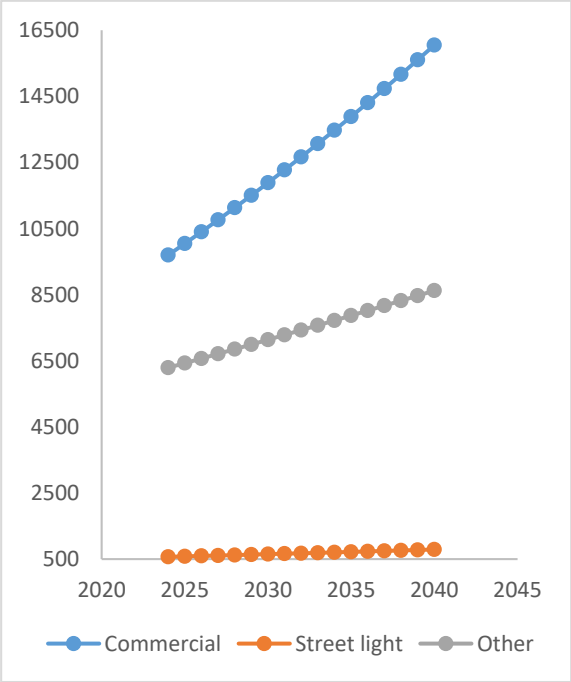

Figure 5 : Graphical anaylsis of forecasted values from 2024-2040 of Commercial,Street-light and Others sectors.

CONCLUSION

Electricity is a critical resource for any country, as it powers homes, businesses, and industries. Pakistan, like many developing nations, faces a significant challenge when it comes to meet the growing demand for electricity. According to the International Energy Agency (IEA), Pakistan's electricity demand has increased by 70% over the past decade, and it is expected to continue to grow in the coming years. Given this trend, it is essential for the government of Pakistan to take effective steps to increase electricity production from a variety of energy sources in order to keep the economy of the country stable by meeting the demand for electricity.

This comprehensive study conducted an in-depth analysis of Pakistan's electricity consumption and aimed to develop a suitable model for forecasting the country's future electricity consumption (E.C.). For this purpose, the electricity consumption, GDP per capita, and population data have been taken and analyzed from 1972 to 2020. Notably, the study showed a strong correlation (0.985, 0.965, 0.985, 0.944, 0.902, 0.938, 0.989) between E.C. in various sectors (household, commercial, industrial, agricultural, streetlight, other) with population. Similarly, the E.C. in these sectors showed a similar correlation (0.965, 0.982, 0.944, 0.869, 0.850, 0.882, and 0.957) with GDP. To predict population and GDP different mathematical model such as linear, Exponential, Polynomial and logarithmic models were applied on population and GDP data. In order to get the best fitted model a number of goodness-of-fit tests Coefficient of Determination, Sum of Square Errors, Sums of Square Regression, Mean Square Error, Root Mean Square Error, And F- Statistics (R^2, SSE, SSR, MSE, RMSE and F Statistics) applied and got the polynomial model to be the most effective one for population and GDP. The models linked EC (electricity consumption) to population and GDP per capita, providing a comprehensive understanding of the demand for electricity across different sectors.

The study also throws light on the anticipated future trends in Pakistan's population, GDP per capita, and electricity consumption, based on the selected models suggested by goodness of fit tests. According to the analysis, Pakistan's

population is expected to reach approximately 244428310 by the year 2025, showing a steady growth pattern. According to our analysis the expected population in 2030 may be approximately more than 268 million. Similarly, in 2035 and 2040 the future population may increase by 293567191 and 319825879 respectively according to the best fitted model as shown in table 4.9. These increasing population results demand from higher authorities and researchers to think over it and consider it in future planning on priority basis.

The study also predicts the future trajectory of Pakistan's GDP per capita on the basis of historical data. As per our analysis in 2025 it is expected to have an estimated value of GDP as 1958.3$ per capita, reflecting potential economic growth of the country. This study also shows that the future GDP per capita in Pakistan will be 2338.2$, 2,755.8$ and 3210.9$ in 2030, 2035 and 2040 respectively.

The study also emphasizes the relationship between electricity consumption with GDP and Population of Pakistan. As the data were not equally spaced, to check the best possible behavior of data we applied divided difference rule on data of electricity consumption. So, we calculated the first divided difference of all sectors of electricity consumption depending upon population and GDP. The approximate constant values for the first divided differences indicated that a linear model would be the most suitable one. That's why we used a Multiple Linear Regression (MLR) model for the analysis of electricity consumption with population and GDP Per Capita. The parameters of MLR have been estimated by using the method of least square. The high value of adjusted R-square (Adj R^2) and the least value of root mean squared error (RMSE) indicate that the sector of electricity consumption is significantly affected or explained by the variables or factors under consideration in our analysis. Our model residual show it is normal distributed. Normally distributed residuals indicate that our model is capturing the main patterns and relationships in the data and suggests that the factors used in this analysis have a strong influence on electricity consumption in all sectors. According to the selected model the expected electricity consumption in the country is estimated to be 121,121.1 GWh (gigawatt-hours) in 2025, 138,861.8 GWh in 2030, 157,716.8 GWh in 2035, and 177,686.3 GWh in 2040. These figures shows that demand of electricity is expected to be grown quickly in the future suggesting that more investments are needed in electricity sector and expand its power infrastructure to keep up with the growing demand.

The implications of these results extend to various sectors, including power management, healthcare, urban planning, and overall development. Incorporating population dynamics and economic growth considerations in long-term electricity planning can contribute to sustainable and efficient electricity management in Pakistan.

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DECLARATION OF COMPETING INTEREST

The authors have no relevant financial or non-financial interests to disclose.

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